UNIVERSITY OF SURREY
DEPARTMENT OF PHYSICS

Level 1 Laboratory: Introduction Experiment

Determination of “g” using a spring

INTRODUCTION

This experiment is designed to get you confident in using the quantitative methods of estimating experimental uncertainty.

LABORATORY DIARIES

• Don’t forget to record all your sketches of the apparatus, measurements, observations, uncertainties, graphs and interpretations directly into your laboratory diary. Do this in “real time” i.e. as you conduct the experiment. Do not censor your diary afterwards. If you make mistakes, simply score through these, giving a few words of explanation.

• Do not write on scraps of paper and later transcribe these results into your diary!

• The diary should be your one and only written record of all experiments you conduct. (Note : the diary is quite distinct from a report which is a much more formal and organised account of your experiment)

AIMS

There are several aims to this experiment:

1. to measure $g$, the acceleration due to gravity on the earth’s surface.
2. to quantify the experimental uncertainty associated with estimating the gradient in straight line graphs.
3. to combine the uncertainties of two different measured quantities to determine the overall uncertainty in the measurement
4. to consider sources of systematic uncertainty in your experimental method.

EXPERIMENT

The experiment has two parts. In the first ‘dynamic’ part, you will measure the oscillation period of a spring as a function of suspended mass and hence determine the spring constant of the spring from the gradient of a graph plotted using your data. In the second ‘static’ part you will measure the ratio of the extension of the same spring as a function of suspended mass to determine the value of $g$, the acceleration due to gravity. Since $g$ is a known value, you will be able to consider using your measured value of $g$ and its uncertainty whether there is a systematic error in your measurement or just random errors.
THEORY

When a mass is suspended from the end of a spring, Hooke’s Law states that the extension of the spring is proportional to the mass, provided the elastic limit of the spring is not exceeded. Generally, the tension force, $T$, in the spring is proportional to the extension $x$ produced. That is

$$T = kx,$$

where $k$ (N/m) is the spring constant.

Consider a spring PA of length $l$ suspended from a fixed point P, Fig. 1. When a mass $m$ is placed on it, the spring stretches to O by a length $e$ given by

$$mg = ke,$$  \hspace{1cm} (i)

since the tension of the spring is then $mg$.

If the mass is pulled down a little and then released, it oscillates up and down above and below O. The tension of the spring at B is then equal to $k(e+x)$, and hence the force towards O is

$$= k(e+x)-mg.$$  

Since force = mass x acceleration,

$$-[k(e + x) - mg] = ma.$$  

The minus indicates that the net force is upwards at this instant, whereas the displacement $x$ is measured from O in the opposite direction at the same instant. From this equation,

$$-ke - kx + mg = ma.$$  

But, from (i),

$$mg = ke,$$

therefore

$$-kx = ma$$

to

$$a = -\frac{k}{m}x = -\omega^2 x,$$

where $\omega^2 = k/m$. Thus the motion is simple harmonic about O, and the period is given by

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}.$$  \hspace{1cm} (ii)

From (ii), it follows that $T^2 = 4\pi^2 m/k$. Consequently a graph of $T^2$ against $m$ should be a straight line through the origin. This theory, however, does not take into account the finite
mass $s$ of the spring. A more advance treatment shows that the period of vibration $T$ is given by:

$$T = 2\pi \sqrt{\frac{m + \lambda s}{k}}$$  

(iii)

where $\lambda$ is approximately $1/3$.

If a mass $M$ is is now placed on the end of the spring, producing a steady extension less than the elastic limit, then $Mg = ke$, therefore

$$g = \frac{e}{M} k .$$  

(iv)

By attaching different masses to the spring, and measuring the corresponding extension, the magnitude of $e/M$ can be found by plotting $e$ against $M$ and measuring the gradient of the line. This is the ‘static’ measurement on the spring. From the magnitude of the ‘dynamic’ measurement when the period was determined from different loads, the value of $g$ can be found by substituting the magnitudes of $e/M$ and $k$ in (iv).

**PART 1  Dynamic measurements**

In this first part you will determine the spring constant from the gradient of a graph plotted using data obtained by measuring the spring period as a function of suspended mass. In this introductory experiment whose purpose is to provide you with data to practice data analysis methods introduced in the tutorial, it is better not to repeat and average any of your individual measurements. This is because averaging will reduce the random scatter of the data about the straight line, and therefore make your use of the parallelogram method for uncertainty estimation less instructive. Complete each part and answer the questions before moving on to the next part.

1. First, suspend your spring and hanger (loaded up to 100g) on the retort stand. Make sure that the weight is carefully released to prevent the spring being stretched beyond its elastic limit. Make notes and drawings of your method and apparatus in your laboratory diary on a clean, dated, titled page, with margin ruled.

2. Set the weight into oscillation by pulling it down a few cm and gently releasing it. Time five oscillations (again, this number is minimized to increase the random uncertainty in your measurement for instructive purposes). Enter your data into a pre-prepared table which include the variables that you are measuring (with their units) and any extra columns that you need to process this raw data to arrive at the data you wish to plot. Don’t forget that the units of a processed variable will be modified in the same way as the variable e.g. if you were to square $T$, its units will be $s^2$). Write your measurements in your table as you take the readings and design your table so that the numbers you read off of your instrument are the numbers that you record, without any intervening arithmetic!

3. Repeat these steps using 10 different masses and complete your table and plot a suitable graph. Draw a line of best fit and determine its gradient using as big a triangle as is possible.
4. Now, apply the “Parallelogram Method” to your graph (described in the Appendix at the end of the script). Using the parallelogram method, do the following:

- Estimate the statistical uncertainty (the “random error”) in the experimental measurements of the spring constant, and add appropriate error bars to your data points.
- Estimate the uncertainty (or “error”) in the gradient and intercept. If zero does not fall within the bounds of your estimated uncertainty consider whether you might have underestimated the random error, or whether the error is systematic (i.e. due to mis-calibration of your measuring instruments, or the parameters that you are accurately measuring are not the same as the variables defined in your hypothesis. Write down your result for the spring constant, including units and experimental uncertainties. Fix your graph into your diary at the appropriate place (remembering to include units, axes labels and giving the graph a title).

PART 2 Static measurements

In this second part you will measure the ratio of the extension (of the same spring!) as a function of suspended mass to determine the value of \( g \), the acceleration due to gravity.

1. Suspend your spring and unloaded hanger on the retort stand and position the metre rule in close proximity to a convenient point on the hanger to measure the extension of the spring. Make notes and drawings of your method and apparatus in your laboratory diary. Enter the reading on the metre rule corresponding to the “start position” of hanger into a pre-prepared table (which again includes all the variables that you are measuring (with their units) and any extra columns that you need to process this raw data to arrive at the data you wish to plot).
2. Add a 10g mass and record the new position of the hanger, and repeat this for the remaining masses. Complete the table then plot a suitable graph to obtain a value for \( e/M \).
3. Draw a line of best fit and determine its gradient (\( e/M \)) using as big a triangle as possible. Determine the uncertainty in the gradient using the parallelogram method, as before, and write down your result, including unit and experimental uncertainty. Fix your graph into your diary at the appropriate place (again, remembering to include units, axes labels and giving the graph a title).
4. Equation (iv) shows that the value of \( g \) is produced by multiplying the values of the gradients of the graphs produced in the dynamic and static experiments. However, the error is the value of \( k \) and \( e/M \) also need to be combined. This may be achieved using the analysis given in the laboratory handbook. In the general case, if \( z = x.y \), then

\[
\frac{s_z}{z} = \sqrt{\left(\frac{s_x}{x}\right)^2 + \left(\frac{s_y}{y}\right)^2},
\]

where the \( s \) values represent the standard errors in the mean value of the variables given in the subscript. A less rigorous analysis gives the alternate result

\[
\left(\frac{s_z}{z}\right) \approx \left(\frac{s_x}{x}\right) + \left(\frac{s_y}{y}\right)
\]
Applying the latter equation to our situation, we simply have to add the fractional errors (that’s the error given by the Parallelogram Method divided by the mean value) of the two gradients to arrive at the total fractional error in \( g \). Write down, showing your workings, your result for \( g \), including units and the experimental uncertainty.

**PART 3  Evaluation**

By this stage you will have determined whether the accepted value for \( g \) (9.81 N/kg) falls within the range of possible values determined by your results. If it does, should you be congratulating yourself? It might be that the excellent estimation of the large uncertainties created by your poorly executed experiment simply hid the existence of second order effects, like the mass of the spring \( s \) mentioned in the theory section, or other sources of experimental error; hardly a cause for celebration!

If you were to repeat and average each reading then your error bars should reduce in size by a factor \( 1/\sqrt{n} \), where \( n \) is the number of repeats (so nine measurements of each data point should reduce the scatter of the data points around your line of best fit by one third). The increased precision provided by this more careful experimental method might then reveal systematic errors which could perhaps be explained by the more sophisticated hypothesis given above. If your data does reveal an apparent systematic error, weigh the spring and estimate whether the correction offered in equation (iii) moves your mean value of \( g \) in the right direction. If your uncertainties are too large to reveal this, then if time allows, repeat and average your data to reduce the uncertainty of your measurements to gain further insight.

This experiment is not finished until you have made a critical review of your results, either using the suggestion given above, or any other ideas that might come to mind. A good scientist will review their experimental method and make suggestions for improvement should they, or others, wish to use their data as a basis for future work.

**Bibliography**

Department of Physics, Level 1 Laboratory Handbook, Lancefield & Murdin

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The Parallelogram Method

Figure 1: The full construction for determining uncertainty in the slope and intercept of a straight-line fit to experimental data, by the Parallelogram Method. The stages in this construction are shown overleaf as (a) - (f).

The series of figures (a) to (f) on the next two pages will help guide you through the application of the Parallelogram Method. The following points should be borne in mind:

- The parallelogram drawn on your graph should be such that it encloses only about **two thirds** of the data points. (Why two thirds?)
- The long sides of the parallelogram should be parallel to your estimated best-fit line.
- The ends of the parallelogram should be by the first and last data points.
- The max. and min. values of the slope ($m_H$ and $m_L$ respectively) and intercept ($c_H$ and $c_L$) are those of the two diagonals of the parallelogram, as shown in Fig. 1.
- The error in the gradient ($\Delta m$) and intercept ($\Delta c$) of your best line should be estimated from the two diagonals using:

  \[ \Delta m = \frac{m_H - m_L}{\sqrt{n}} \quad ; \quad \Delta c = \frac{c_H - c_L}{\sqrt{n}} \]

  , where $n$ is the total number of data points.

  Why should there be a factor of $1/\sqrt{n}$ in these expressions for the error?

- The error bars on your experimental data points should be big enough so that about **two thirds** of them touch or cross your best-fit line. This means, therefore, that the total length of the error bars will be the same as the height of the parallelogram.
(a) In this example (same data as in Fig. 1), there is an increase of $y$ as $x$ is increased. It appears roughly linear but there is significant random error in the data.

(b) We draw the best straight line by eye, fitting to the experimental data as best we can. The line should lie along the “centre of mass” of the points so that the points above the line, taking into account their vertical distance away from it, are “balanced” as near as possible by the equivalent points below the line. In otherwords, we are crudely trying to minimise the sum of the deviations of the points from the line.

(c) We draw the parallelogram formed by translating the best fit line upwards and downwards, so that the parallelogram encloses about $2/3$ of the experimental points.

For random experimental errors and for a sufficient number of data points, the best-fit line will be translated upwards and downwards by about the same amount.
(d) We draw the lines of minimum and maximum slope, via the 2 diagonals of the parallelogram.

We then extrapolate back to the y-axis to yield the highest and lowest values of the gradient and intercept.

(e) Now we draw in error bars of the same height as the parallelogram.

Generally, this will result in about 2/3 of them touching the line of best fit.

(f) In serious scientific publications the parallelogram and extreme lines are sometimes omitted as being understood (see left.)

However, we strongly recommend that, in your diary and reports, you always show the extreme lines (and parallelogram) as in Fig. 1, if you have used these to determine the uncertainty in the slope and/or intercept.

Uncertainties should always be quoted when you quote a slope or intercept determined from experimental data.